4-1 Notes: Antiderivatives and Indefinite Integration Name _____
Calculus AB

Example 1) Find the function F whose derivative is $f(x) = 5x^4$.

Definition of Antiderivative

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I

Representation of Antiderivatives

- If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form $G(x) = F(x) + C$, for all x in I where C is a constant.
- $G(x)$ is called the "general solution" of the differential equation (equation that involves derivatives of a function)

Example 2) Find the general solution of the differential equation $y' = \frac{1}{2}$.

Notation for Antiderivatives

The operation of finding the general solution to a differential equation is called antidifferentiation OR indefinite integration.

$$y = \int f(x)\,dx = F(x) + C$$

Remember: $\frac{dy}{dx} = f'(x) \Rightarrow dy = f'(x)\,dx$

$$\int dy = \int f'(x)\,dx$$

$$y = f(x) + C$$

Note: integration "undoes" differentiation (antidifferentiation)

Basic Integration Rules

Differentiation Formula	**Integration Formula**
$\frac{d}{dx}[C]=0$	$\int 0\,dx=C$
$\frac{d}{dx}[kx]=k$	$\int k\,dx=k\,x+C$
$\frac{d}{dx}[k\,f(x)]=k\,f'(x)$	$\int k\,f(x)\,dx=k\int f(x)\,dx$
$\frac{d}{dx}[f(x)\pm g(x)]=f'(x)\pm g'(x)$	$\int[f(x)\pm g(x)]dx=\int f(x)\,dx\pm\int g(x)\,dx$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq -1$ *Power Rule*
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx=\sin x+C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx=-\cos x+C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx=\tan x+C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x\tan x\,dx=\sec x+C$
$\frac{d}{dx}[\cot x]=-\csc^2 x$	$\int \csc^2 x\,dx=-\cot x+C$
$\frac{d}{dx}[\csc x]=-\csc x\cot x$	$\int \csc x\cot x\,dx=-\csc x+C$

Example 3) $\int \frac{1}{x\sqrt{x}}dx$

Example 4) $\int x(x^2+3)\,dx$

Example 5) $\int\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)dx$

Example 6) $\int\left(\frac{x^2+1}{x^2}\right)dx$

Example 7) $\int\left(t^2-\sin t\right)dt$

Example 8) $\int\sec y\left(\tan y-\sec y\right)dy$

Example 9) $\int \frac{\sin x}{1-\sin^2 x}\,dx$

Example 10) $\int \sqrt[3]{x}(x-4)\,dx$

Example 11) Find the equation of y given the derivative and the indicated point on the curve. $\frac{dy}{dx} = 2(x-1)$ (3, 2)

Example 12) Find the equation of y given the derivative and the indicated point on the curve. $\frac{dy}{dx} = -\frac{1}{x^2}$ (1, 3)

Example 13) Solve the differential equation.

$$f''(x) = x^2, \quad f'(0) = 6, \quad f(0) = 3$$

Example 14) A baseball is thrown upward from ground level with a velocity of 10 meters per second. Determine its maximum height. (use $a(t) = -9.8 m/\sec^2$)

Example 15) A car traveling at 45 miles per hour is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied.

a) How far has the car moved when its speed has been reduced to 30 miles per hour?

b) How far has the car moved when its speed has been reduced to 15 miles per hour?